

An important method for comparing and describing the relationship between two quantities is by using ratios. Ratios are used in expressing how fast a jeep travels (i.e. 60 kilometers/hour), how much a certain amount of a product costs (i.e. P 40/ kilo) or how fast a secretary types (i.e. 60 words per minute).

Proportions on the other hand are used to estimate the measurements of big sizes or large distances like the width of a river or the height of very tall objects. It is also used to solve problems on rates (i.e. cost of an overseas call or the speed of a moving car). In this module, you will learn about ratio and proportion.

This module is divided into two lessons:

Lesson 1 – Learning about Ratios

Lesson 2 – Learning about Proportions

## What Will You Learn From This Module?

After studying this module, you should be able to:

- explain the meaning of ratio;
- simplify a ratio to its lowest term;
- identify equivalent ratios;
- differentiate between ratio and rate;
- solve word problems involving rates;
- explain the meaning of proportion; and
- solve daily life problems involving ratio and proportion.



Before studying this module, you should have studied the modules entitled *Multiplication and Division of Fractions* and *Multiplication and Division*.



Before starting with the lessons of this module, answer the following test items first. This will determine what you already know about the topic.

- A. 1. An NFE Facilitator found that there are 46 males and 54 females in all his sessions. Find the ratio of the following.
  - a. The ratio of males to females.
  - b. The ratio of females to the entire population of learners.
  - 2. Give two equivalent ratios for the following. Solve for and encircle the simplest form for each ratio. (2 points each)
    - a. 4:10
    - b. 15:30
  - 3. Find the rate of the following.
    - a. An 8-minute overseas call costs P164.00. What is the cost of the call per minute?

b. A secretary can type 220 words in 4 minutes. How fast can she type in one minute?

- B. Solve for the following problems using ratio and proportion.
  - 1. In a medical mission, doctors determined that there were 2 out of 25 people afflicted with tuberculosis in a barrio. If there were 600 people living in the town, how many people had tuberculosis?

2. If 200 sheets of bond paper costs P 50.00, how much will 500 sheets cost?

C. Write true if the ratios are proportional and write false if they are not.

 1.	15:7=5:2
 2.	7:9=14:18
 3.	4: 3 = 24 : 18

Well, how was it? Do you think you fared well? Compare your answers with those in the *Answer Key* on pages 35–37.

If all your answers are correct, very good! This shows that you already know much about the topics in this module. You may still study the module to review what you already know. Who knows, you might learn a few more new things as well.

If you got a low score, don't feel bad. This means that this module is for you. It will help you understand important concepts that you can apply in your daily life. If you study this module carefully, you will learn the answers to all the items in the test and a lot more! Are you ready?

You may now go to the next page to begin Lesson 1.

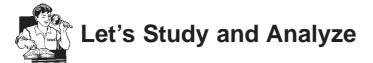
### LESSON 1

### **Learning About Ratios**

In this lesson we will study about ratios. We use ratios to compare two quantities that are somehow related to each other. So we may compare the distance traveled by a motorcycle by the time it takes to cover the distance (i.e. kilometers traveled per hour) or we may compare the population of males to females in a certain group.

In this lesson we will learn about ratios and how this can be applied to everyday problems. It is important to first learn about *ratios* in order to understand *proportion*. After studying this lesson, you should be able to:

- identify/find the ratio of two given quantities;
- simplify a given ratio into its lowest term;
- identify the equivalent ratio/s of a given ratio;
- differentiate between ratio and rate;
- identify/find the rate of two given quantities;

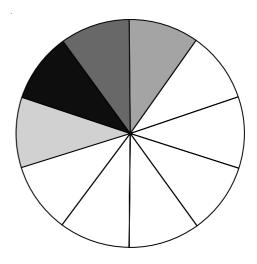


How do we compare two quantities? A ratio is a comparison by division of two quantities of the same kind and in the same unit. The result of the comparison is a number without unit. Let us take a look at some situations on the next page so that you will understand the concept of ratio more clearly.

#### Situation 1

Look at the circular figure at the right. How many shaded parts are there? How many parts are there in all?

What is the ratio of the shaded part to the whole figure? The ratio is: 4 shaded parts to the 10 parts of the whole figure, written in fraction form as 4/10; or in ratio form as 4:10, read as "4 is to 10". The colon (:) is used to express the ratio of two quantities.



What is the ratio of the whole figure

to the shaded parts? The ratio is: 10 parts of the whole figure to the 4 shaded parts, written in fraction form as 10/4 or in ratio form 10:4, read as "10 is to 4."

### Situation 2

In a Nonformal Education Learning Center, there are 10 male learners and 15 female learners in one learning group. What is the ratio of the male learners to the female learners?

The ratio is: 10 male learners to 15 female learners written in fraction form 10

as  $\frac{10}{15}$  or in ratio form as 10:15, read as "10 is to 15".

What is the ratio of the female learners to the male learners? The ratio is:

15 female learners to 10 male learners written in fraction form as  $\frac{15}{10}$  or in ratio form as 15:10, read as "15 it to 10".

### Situation 3

When the quantities are of the same kind but given in different units, we have to express them in the same units.

Let's analyze this problem.

Jose took 18 days to finish his project while Romy took 3 weeks. What is the ratio the length of time it took the two boys to finish their project?

First, we have to change 3 weeks to 21 days.

Thus: Jose's time: Romy's time = 18 : 21. This means that the ratio of the length of time it took the 2 boys to finish their project is 18 : 21. (Simplifying ratio to its lowest form will come in the next section of this lesson, all these examples will be used in that particular section later.)

#### Situation 4

A tree is 3 meters high. A bamboo pole is 150 cm. long. What is the ratio of the height of the tree to the length of the bamboo pole?

This problem illustrates the need to use the same units. Thus, we first change 3 meters to 300 cm.

 $\frac{\text{height of the tree}}{\text{height of the pole}} = \frac{300}{150} \text{ or } 300:150$ 

Suppose we change 150 cm to meters instead. Will we get the same ratio?

Let us try: How many meters is equivalent to 150 cm?

$$150 \text{ em} \times \frac{1 \text{ m}}{100 \text{ em}} = 1.5 \text{ m}$$

Therefore, we have:

$$\frac{\text{height of the tree}}{\text{height of the pole}} = \frac{3}{1.5} \text{ or } 3:1.5$$

Did we get the same ratio?

In each of the situations, two quantities were compared by division. The answer in the comparison is called the ratio of the two quantities. Notice that the ratio in the different situations had no unit.



A. Write the following as a ratio in fraction form and in another form using a colon (:).The first one is done for you.

1.	25 learners to 1 Instructional Manager	$\frac{\underline{25}}{\underline{1}}$	25:1
2.	6 tablet chairs to 6 learners		
3.	6 lessons to 2 modules		
4.	5 learning centers to 10 Instructional Managers		
5.	8 hours of sleep to 24 hours		

B. Find the ratio of the first quantity to the second quantity. Make sure the two quantities are first expressed in the same unit. The first one is done for you.

1.	6 cm to 2 dm	=	6:20
2.	15 days to 2 weeks	=	
3.	5 dm to 1 meter	=	
4.	15 hrs to 1 day	=	
5.	2,300 m to 5 km	=	

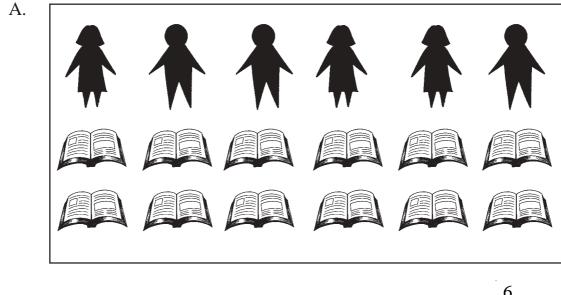
Compare your anwer with the Answer Key on page 38.



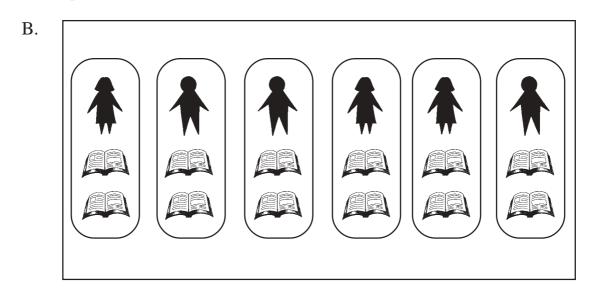
### **Ratio in Simplest Form**

Since ratios are also fractions, they can be expressed in their simplest form or lowest term.

Look at the learners and the modules.



The ratio of the numbers of learners to the number of modules is  $\frac{6}{12}$  or 6:12. Group the same number of learners and modules in another way.



You can say that the ratio of the number of learners to the number of modules is  $\frac{1}{2}$  or 1:2.

The two ratios describe the same comparison but ratio A is expressed in its simplest form in ratio B. Therefore, in the above example,  $\frac{6}{12}$  or 6 : 12 in

its simplest form is  $\frac{1}{2}$  or 1:2.

A ratio described by a pair of numbers with only 1 as the common factor is a ratio in its simplest form.

By computation, how do we find the simplest form of a given ratio?

Recall that in reducing or simplifying fractions to lowest term, we divide the numerator and the denominator by their greatest common factor (GCF).

Similarly in simplifying ratio to its lowest term, divide both numbers by their GCF.

In Example A, the ratio is 6 : 12 and their GCF is 6. Thus, we have:

$$\frac{6:12}{6} = 1:2$$
  
or  
 $6:12 \div 6 = 1:2$ 



Let's take a look again at the ratio of the two quantities in Situation 1 on page 3. The ratio is 4 shaded parts to the 10 parts of the whole figure or 4:10.

Simplify the following ratio to their lowest term. The first one is done for you.

1. 
$$\boxed{4}:10 \rightarrow 4:10 \div \boxed{2} = 2:5$$

Use the ratio in Situation 2 on page 5.

2. 
$$\square: \square \rightarrow \_\_\_ \div \square = \_\_\_$$

Use the ratio in Situation 3 on page 5.

3.  $\square:\square \rightarrow \_ \div \square = \_$ Use the ratio in Situation 4 on page 6 4. a.  $\square:\square \rightarrow \_ \div \square = \_$ b.  $\square:\square \rightarrow \_ \div \square = \_$ 

Compare your answer with the Answer Key on page 38.

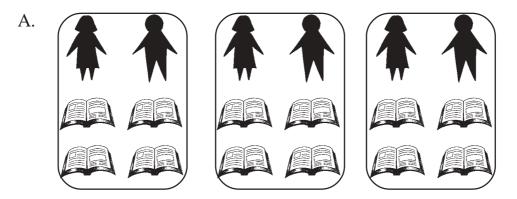


### **Equivalent Ratio**

Let's look back at the number of learners and the number of modules on page 8. The ratios 6 : 12 and 1 : 2 both describe the comparison of the number of learners and the number of modules.

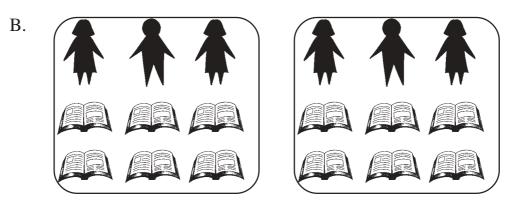
Suppose we group the same number of learners and modules in different ways.

Example 1



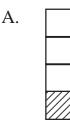
You can also say that the ratio of the number of learners to the number of modules is 2:4.

The ratios shown on page 9 (6:12 and 1:2) and the ratios shown on page 10 and this page (2:4 and 3:6) are **equivalent ratios**. They describe the same pair of groups of learners and modules. The simplest form is 1:2.



### Example 2

Let's look at another example of equivalent ratios.

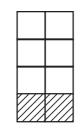


B.

С.

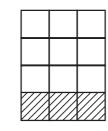
What is the ratio of the shaded part to the unshaded parts? It is 1:3.

Let's have an equivalent ratio of figure A. Let's take a look at figure B.



What is the ratio of the shaded part to the unshaded part? It is 2:6.

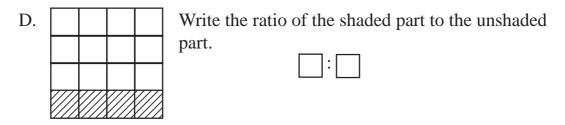
Let's have a third equivalent ratio.





If your answer is 3:9, then you're correct!

Let's have a fourth one.



If your answer is 4:12, then you're correct!

And we can have more equivalent ratios. You can say that 1:3, 2:6, 3:9, and 4:12 are equivalent ratios. The simplest form is 1:3.

By computation, how do we get the equivalent ratio of a given ratio.

To get the equivalent ratio/s of a given ratio, multiply or divide both numbers of the ratio by the same number.

Thus, in Example 1,

$$6:12 \quad \rightarrow \quad \frac{6:12}{6} = 1:2$$

$$6:12 \quad \rightarrow \quad \frac{6:12}{6} = 2:4$$

$$6:12 \quad \rightarrow \quad \frac{6:12}{6} = 3:6$$

In Example 2,

1:3	$\rightarrow$	$2 \times 1:3 = 2:6$
1:3	$\rightarrow$	$3 \times 1:3 = 3:9$
1:3	$\rightarrow$	$4 \times 1:3 = 4:12$



A. Give two equivalent ratios for each of the following:

1.	1:5	 
2.	10:20	 
3.	2:3	 
4.	8:4	 
5.	4:16	 

B. From the three sets of equivalent ratios below, encircle the ratio in its simplest form.

1.	4:20	1:5	3:15
2.	5:10	3:15	1:2
3.	2:3	8:12	10:15
4.	24:12	2:1	8:4
5.	4:16	2:8	1:4

Compare your answers with those in the Answer Key on pages 38–39.

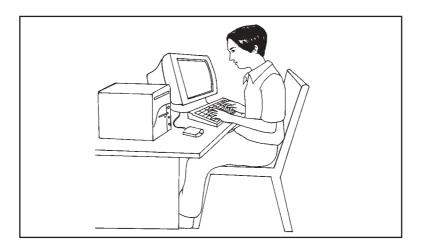
# Let's Study and Analyze

When we compare two quantities of different kinds by division, we use the term **rate** instead of ratio.

Let's study and analyze the following problems to help you understand the concept of rates.

### EXAMPLE 1

Rolly can type 300 words in 5 minutes. About how many words a minute can he type?



In this problem, we are comparing the number of words to the number of minutes (time) it takes to type the words.

**STEP 1** Solution:

 $\frac{300 \text{ words}}{5 \min} = 60 \text{ words/min}$ 

This means that the rate of Rolly's typing is 60 words.

**STEP 2** Simplify the ratio.

$$300:5 \longrightarrow \frac{300:5}{5} = 60:1$$

This means that the rate Rolly can type 60 words per minute.

Notice that the first term (60) is expressed in *words* while the second term (1) is expressed in *minutes*. This rate can be written in symbols as *60 words/ minute*. This example deals with the rate expressed as quantity over time.

Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$

Examples of these are 50 kilometers per hour (50 km/hr.) 10 meters per second (10 m/s), 7 liters per minute (7L/min).

- **EXAMPLE 2** 3 kilos of rice cost P60.00. What is the unit price of the rice?
- **SOLUTION** In this problem, we are again comparing quantities of different kinds: the cost and quantity of a product. Hence, we are computing the rate.

$$\frac{\mathbf{P}\,60}{3\,\mathbf{k}} = \mathbf{P}\,20/\mathbf{k}$$

The ratio of the amount in pesos to the amount in kilos is 20 is to 1. This means that the rice costs P20.00 per kilo.

Notice that the first term (20) is expressed in pesos while the second term (1) is expressed in kilos. This rate can be written in symbols as **P20.00/kilo**.

The second example deals with a rate expressed as cost over quantity.

Rate =  $\frac{\text{Cost}}{\text{Quantity}}$ 

Examples of these are ₱15.00/kilo, ₱205.00/box, ₱59.50/liter, etc.



Find the rate of the following.

1. A 50-kilo sack of rice costs ₱900.00. What is the price per kilo?

2. Bong can finish the 100-meter dash in 20 seconds. How many meters can he cover per second?

3. 7 liters of gasoline costs ₱119.00. What is the price per liter?

4. Joy can type 325 words in 5 minutes. How many words can she type per minute?

5. A jeep traveled 161 kilometers in 3.5 hours. What is its speed per hour?

Compare your answers with those in the Answer Key on pages 40-41.



- 1. Bong has 12 coins and Jun has 20 coins.
  - a. Give the ratio of Bong's coins to Jun's.
  - b. Give the ratio of Jun's coins to Bong's.
  - c. Find the simplest form of the ratio and give two equivalent ratios of the ratio of Bong's coins to Jun's. Encircle the ratio in simplest form (3 points)
  - d. Give two equivalent ratios of Bong's coins to the difference of his coins and Jun's. Encircle the ratio in simplest form. (3 points)

- 2. Find the rate of the following. (1 point each)
  - a. 5 liters of milk costs  $\mathbb{P}200.00$ . What is the price per liter?
  - b. Lolit can weave 52 baskets in 13 days. How many baskets can she weave per day?
  - c. 6 kilos of beef costs P690.00. What is the cost per kilo?
  - d. A bus traveled 640 km in 8 hrs. What is its average speed?

Compare your answers with those found in the *Answer Key* on pages 41–43.

If your test score is from:

- 7–8 Excellent! You have understood the lesson well.
- 5–6 Review the parts of the module which you did not understand.
- 0–4 You should study the whole lesson again.

If you did well on this test, congratulations! You may now go to the next lesson.



- A ratio is a comparison by division of two quantities of the same kind and in the same unit.
- The colon (:) is used to express the ratio of two quantities, e.g. 1:2
- A ratio is simplified into its lowest term by dividing both numbers by their greatest common factor.
- A ratio is in its simplest form if and when the pair of numbers have only 1 as the common factor.
- Equivalent ratios describe the same pair of group of quantities, e.g. 1:2, 2:4, 3:6, 4:8, 5:10 etc. with one of the ratios in its simplest form. In this case, it is 1:2.
- To get the equivalent ratio/s of a given ratio, multiply or divide both numbers by the same number.
- When two quantities of different kinds or names are compared by division, the term rate is used instead of ratio.
- There are two general types of rates:
- The first one deals with a ratio expressed as *quantity over time*.

Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$

• The second deals with a ratio expressed as *cost over quantity*.

Rate =  $\frac{\text{Cost}}{\text{Quantity}}$ 

### LESSON 2

### **Learning About Proportion**

Now that you have understood what ratios are and how they are used, we can move on to proportions. Proportions are used in many practical applications like estimating the length of long distances or the length of very tall buildings. Proportion is also used in adjusting the amount of ingredients in a recipe when you prepare either more or less servings than the standard recipe.

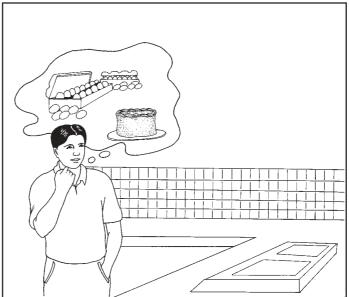
After studying this lesson, you should be able to:

- read and write proportions in symbols and words; and
- solve word problems involving proportion.



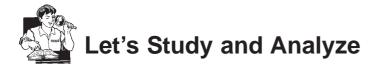
Joseph is an apprentice cook in a restaurant. He had been asked to do simple tasks such as preparing simple recipes and cleaning the kitchen area. One day, the assistant cook got sick and failed to report for work. It was a weekend and the restaurant was full. There were several orders and the chief cook couldn't prepare all of them. He asked Joseph to prepare the orders of some of the customers.

There was an order for 20 hotcakes. A recipe needs 3 eggs to make 5 hotcakes. Joseph is asked to prepare the eggs for 20 hotcakes. He has to determine how many eggs he will need such that the quality of the 20 hotcakes is the same as the 5 hotcakes.



Do you have any idea how Joseph should go about computing the number of eggs to be used? To solve the problem, you have to use ratio and proportion.

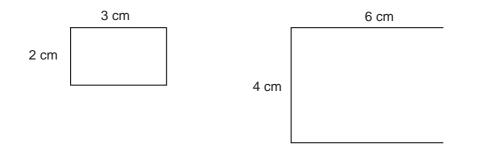
By using ratio and proportion Joseph was able to determine that he needed 12 eggs to make 20 hotcakes. But how did Joseph arrive at that answer? Read on to find out.



In Lesson 1, we have learned how to identify equivalent ratios. You will make use of that skill in this lesson. Two equivalent ratios form a proportion such as 1 : 2 = 2:4 read as "one is to two, as two is to four or in fraction form,

 $\frac{1}{2} = \frac{2}{4}$ . Can you name some more pairs of equivalent ratios?

When do we say that two ratios are proportional to each other? Let us take a look at the two rectangles below. Their lengths and widths are given.



Let us take a look at the ratio of the length to the width of the two rectangles.

- The ratio of the small rectangle is 3:2 or  $\frac{3}{2}$
- The ratio of the big rectangle is 6:4 or  $\frac{6}{4}$

For the mean time, focus your attention on the fraction form. If you divide the numerator by the denominator of both ratios, what do you observe?

1.5	1.5
2)3.0	$4 \overline{)6.0}$
2	_4
10	20
1 0	20
$\overline{0}$	0

The quotients of both ratios is 1.5. Therefore, we can say that:

$$\frac{3}{2} = \frac{6}{4}$$
  
or  $3:2 = 6:4$ 

Since the ratios of the length to the width of the two rectangles are equal, then we can say that the two rectangles are *proportional*.

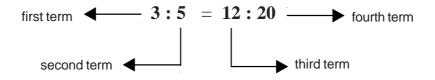
# Let's Learn

Using the example of the hotcakes, 3 eggs makes 5 hotcakes and 12 eggs makes 20 hotcakes. Expressing this as a proportion, we have:

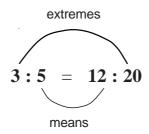
$$\frac{3}{5} = \frac{12}{20}$$
  
or 3:5 = 12:20

It is read as 3 is to 5 as 12 is to 20.

A proportion has four terms as labeled below.



The first and fourth terms (the outer numbers) are called *extremes* while the second and third terms (the inner numbers) are called *means*.



Let us assign a letter of the alphabet for each of the terms. Thus, we have:

$$3:5 = 12:20$$
  
 $a:b = c:d$ 

Using the letters, which are the extremes and which are the means? The outer letters, a and d, are the extremes and the inner letters, b and c, are the means.

One way to check if two ratios form a proportion is to multiply the means and multiply the extremes. The product of the means should be equal to the product of the extremes. This can be written in a general equation:

$$a:b = c:d$$
$$a \times d = b \times c$$

where: a and d are the *extremes* b and c are the *means* 

Let's check if Joseph was correct when he decided that he needed 12 eggs to make 20 hotcakes. The proportion was:

Using the formula: 3:5 = 12:20 $3 \times 20 \stackrel{?}{=} 5 \times 12 (\stackrel{?}{=} \text{ is read as, "should be equal to"})$ 60 = 60

Therefore, the two ratios are proportional.



Check if the following pairs of ratios are proportional. Put a check 4 on the line before each item number if proportional and an 8 if they are not. The first one is done for you.

```
----- 1. 2:3 = 6:9
2 \times 9 \stackrel{?}{=} 3 \times 6
18 = 18
---- 2. 4:5 = 12:20
---- 3. 3:12 = 7:20
4. 5:2 = 15:6
```

Compare your answers with those in the Answer Key on page 43.

## Let's Study and Analyze

In problem solving, one of the four terms is unknown or missing.

Before we move on to problem solving using ratio and proportion, we need to practice solving for the missing number in a proportion. To find one missing number in a proportion, use these formulae:

 $\downarrow$ 

٠	$\Box: b = c: d$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{d} = \mathbf{a}$
٠	$a : \square = c : d$	$(a \times d) \div c = b$
٠	$a : b = \Box : d$	$(a \times d) \div b = c$
٠	$a : b = c : \square$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$

Then, you can check your answer by using the general formula:

 $a \times d = b \times c$ 

Let us apply the formula for computing the missing number in a proportion according to the different situations.

#### Situation 1

A recipe needs 3 eggs to make 5 hot cakes. If Joseph was asked to make 20 hotcakes, how many eggs does he need?

In solving this problem, let's follow the steps below:

**STEP 1** Identify the ratio in words:

the number of eggs to the number of hotcakes.

**STEP 2** Write down the proportion in numbers,

$$3:5 = \square:20$$
$$a:b = c:d$$

**STEP 3** Determine which formula to use in finding the missing number in the proportion.

$$(\mathbf{a} \times \mathbf{d}) \div \mathbf{b} = \mathbf{c}$$

**STEP 4** Substitute the values in the formula and solve for the missing number.

$$3 \times 20 \div 5 = \square$$
$$60 \div 5 = \boxed{12}$$

Therefore, the missing number to form a proportion is  $\boxed{12}$ . The proportion is:

**STEP 5** Check your answer by using the general formula:  $a \times d = b \times c$ 

$$3 \times 20 = 5 \times 12$$
$$60 = 60$$

#### Situation 2

How many days will it take a dressmaker to sew a dozen dresses if it takes her 2 days to sew 3 dresses?

**STEP 1** Identify the ratio in words.

the number of days to the number of dresses sewn

**STEP 2** Write down the proportion in numbers.

$$\Box: 12 = 2:3$$
  
a : b = c : d

**STEP 3** Determine which formula to use in finding the missing number in the proportion.

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{d} = \mathbf{a}$$

**STEP 4** Substitute the values in the formula and solve for the missing number.

$$(12 \times 2) \div 3 = \square$$
$$24 \div 3 = \boxed{18}$$

Therefore, the missing number that forms the proportion is 8. The proportion is:

**STEP 5** Check your answer by using the general formula:  $a \times d = b \times c$ 

$$8 \times 3 \stackrel{?}{=} 12 \times 2$$
$$24 = 24$$

### Situation 3

Most maps are drawn to scale. This means that the distance between any two places on the maps is *proportional* to their real distance.

In a map with a scale of 2 cm to 5 km, the distance between Manila and Biñan is 6 cm. How many kilometers is it between the two places?

**STEP 1** Identify the ratio in words.

a scale of cm to km.

**STEP 2** Write down the proportion in numbers.

$$2:5=6:$$
  
 $a:b=c:d$ 

**STEP 3** Determine which formula to use on finding the missing number in the proportion.

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$$

**STEP 4** Substitute the values in the formula to solve for the missing number.

$$(5 \times 6) \div 2 = \square$$
  
$$30 \div 2 = \boxed{15}$$

Therefore the missing number in the proportion is  $\boxed{15}$ . The proportion is:

$$2:5\stackrel{?}{=}6:15$$

**STEP 5** Check your answer by using the general formula:  $a \times d = b \times c$ 

$$2 \times 15 \stackrel{?}{=} 5 \times 6$$
$$30 = 30$$



Solve the following problems using ratio and proportion. Check your answer also. The first one is done for you.

1. Miss Reyes needs to type a 420 page manuscript. She can type 15 pages in 2.5 hours. In how many hours can she finish typing the manuscript?

$$420 : \Box = 15 : 2.5 (420 \times 2.5) \div 15 = \Box$$

$$4 2 0 \\
\times 2.5 \\
\hline
2 1 0 0 \\
8 4 0 \\
\hline
1050 \div 15 = 70 \\
420 : [70] = 15: 2.5 \\
\hline
15)1050 \\
\hline
105 \\
0
\end{array}$$

Check:

 $420 \times 2.5 \stackrel{?}{=} 70 \times 15$ 1050 = 1050

It will take Mrs. Reyes 70 hrs to type the manuscript.

2. Alex earns ₱700 in selling newspapers a week. How many weeks will it take him to earn ₱3,500.

3. If a 50-kilo sack of rice costs ₱900, how much will 27 kilos of rice cost?

Compare your answers with those in the Answer Key on pages 43-44.



Solve the following problems using ratio and proportion. Solve the problems step by step. (5 points each) - 1 point for every correct step

1. A recipe for making 24 cookies needs 200 grams of flour. How many grams of flour are needed for making 36 cookies?

2. In a map with a scale of 2 centimeters is to 3 kilometers, the distance between two towns is 5 centimeters. How many kilometers is it between the two towns?

3. A survey showed that 3 people preferred brand A for every 5 people that preferred brand B. If a total of 27 people preferred brand A, how many people preferred brand B?

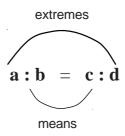
Compare your answers with those in the Answer Key on pages 44-47.

If your test score is from:

- 11–15 Excellent! You have understood the lesson well.
- 7–10 Review the parts of the lesson which you did not understand.
- 0–6 You should study the whole lesson again.

## Let's Remember

- Two equivalent ratios form a **proportion**.
- A proportion has four terms as labeled below. Let a, b, c and d represent numerical values.



- The first and fourth terms are called *extremes* while the second and third terms are called *means*.
- Two ratios form a proportion if the product of the means is equal to the product of the extremes. This can be written in a general equation.

$$a \times d = b \times c$$

where: a and d are the extremes b and c are the means

• To find the missing number in a proportion, use the following formula for each corresponding situation:

Situation	Formula
$\Box: b = c: d$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{d} = \mathbf{a}$
a : $\Box = c : d$	$(a \times d) \div c = b$
a : b = : d	$(a \times d) \div b = c$
$a : b = c : \square$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$

• The steps to follow in solving problem involving ratio and proportion are as follows:

STEP 1	Identify the ratio in words.
STEP 2	Write down the proportion in numbers
STEP 3	Determine the formula to be used in finding the missing number
STEP 4	Substitute the values in the formula to solve for the missing number
STEP 5	Check your answer by using the general equation:

ek your answer by using the gene

 $a \times d = b \times c$ 



### Let's Sum Up

- A ratio is a comparison by division of two quantities of the same kind and in the same unit.
- The colon (:) is used to express the ratio of two quantities.
- A ratio is simplified into its lowest term by dividing both numbers by their greatest common factor.
- A ratio is in its simplest form if and when the pair of numbers have only 1 as the common factor.
- Equivalent ratios describe the same pair of quantities, e.g. 1:2, 2:4, 3:6, 4:8, 5:10 etc. with one of the ratios in its simplest form in this case 1:2.
- To get the equivalent ratio/s of a given ratio, multiply or divide both numbers by the same number.
- When two quantities of different kinds or names are compared by division, the term rate is used instead of ratio.
- There are two general types of rates:
  - The first one deals with a ratio expressed as *quantity over time*.
  - The second deals with a ratio expressed as *cost over quantity*.

- Two equivalent ratios form a **proportion**.
- A proportion has four terms as labeled below. Let a, b, c and d represent numerical values.

$$\mathbf{a}:\mathbf{b} = \mathbf{c}:\mathbf{d}$$

- The first and fourth terms are called *extremes* while the second and third terms are called *means*.
- Two ratios form a proportion if the product of the means is equal to the product of the extremes. This can be written in a general equation.

$$a \times d = b \times c$$

where: a and d are the extremes b and c are the means

• To find the missing number in a proportion, use the following formula in each corresponding situation:

Situation	Formula
$\Box: b = c: d$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{d} = \mathbf{a}$
a : $\Box = c : d$	$(a \times d) \div c = b$
a : b = : d	$(a \times d) \div b = c$
$a:b=c:\square$	$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$

• The steps to follow in solving problems involving ratio and proportion are as follows:

STEP 1	Identify the ratio in words.
STEP 2	Write down the proportion in numbers
STEP 3	Determine the formula use in finding the missing number
STEP 4	Substitute the values in the formula and solve for the missing number
STEP 5	Check your answer by using the general equation:
	$a \times d = b \times c$



- A. 1. In a survey, 36 respondents said they are satisfied with the performance of their Barangay officials while 64 respondents said they are dissatisfied with their performance. Find the ratio of the following. Simplify your answers to lowest terms.
  - a. Find the ratio of the number of satisfied respondents to the number of dissatisfied respondents. (1 point)

b. Find the ratio of the number of dissatisfied respondents to the number of satisfied respondents. (1 point)

c. Find the ratio of the number of dissatisfied respondents to the total number of respondents. (1 point)

d. Find the ratio of the number of satisfied respondents to the total number of respondents. (1 point)

- 2. Give two equivalent ratios for the following.
  - a. 6:15
  - b. 20:25
  - c. 8:1
- 3. Find the rate of the following. (1 point each)
  - a. 3 kilos of mangoes costs ₱72.00.

b. A secretary can type 390 words in 6 minutes.

c. A 7-minute long distance call from Manila to Cebu costs ₱66.50.

d. A motorcycle traveled 117 meters in 9 seconds.

- B. Solve the following problems using ratio and proportion.(5 points for each problem; 1 point for every step done correctly)
  - 1. The standard mixture of cement and sand for plastering purposes is 1 : 3. If Mang Pedro has 4.5 buckets of cement, how many buckets of sand does he need?

2. In a class, 4 out of 5 students passed the math exam. If there are 40 students in the class, how many students passed the exam?

3. Aling Nida can make 3 dresses out of 7.5 meters of clothing material. How many dresses can she make out of 20 meters of clothing material?

Compare your answers with those in the Answer Key on pages 47–53.

If your test score is from:

- 14–17 Excellent! You have understood the lessons of the module well.
- 9–13 Review the lessons in the module which you did not understand.
- 0–8 You must study the whole module again.



### A. Let's See What You Already Know (pages 2–3)

A. 1. a. 
$$46:54$$
 or  $46/_{54}$ 

- b. 54 : 100 or  ${}^{54}/_{100}$
- 2. Any two of the underlined ratios are correct answers. The ratios in simplest form are encircled.

a.	4:10	$\rightarrow$	$\frac{4:10}{2}$	=	2:5 -	in its simplest form. Cannot be
	4:10	$\rightarrow$	$2 \times (4:10)$	=	8:20	reduced further.
	4:10	$\rightarrow$	$3 \times (4:10)$	=	12:30	
b.	15:30	$\rightarrow$	$\frac{15:30}{5}$	=	3:6	

$$\rightarrow \frac{15:30}{15} = \underbrace{1:2} \longrightarrow \text{ in its simplest form. Cannot be reduced further.}} \\ \rightarrow 2 \times (15:30) = 30:60$$

3. a. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

$$164:8 \text{ or } \frac{164}{8}$$

**STEP 2** Solve for the rate.

Rate = 
$$\frac{\text{Cost}}{\text{Quantity}}$$

$$\frac{20.50}{8)164.00}$$

$$\frac{16}{40}$$

$$\frac{40}{0}$$

Therefore, the cost of the overseas call is P20.50 per *minute*.

b. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

220:4 or 
$$\frac{220}{4}$$

**STEP 2** Solve for the rate.

$$Rate = \frac{Quantity}{Time}$$

$$4)220$$

$$20$$

$$20$$

$$20$$

$$0$$

Therefore, the secretary can type 55 words per minute.

B. 1. Solution:

$$2:25 = \square:600$$

$$(2 \times 600) \div 25 = \square$$

$$1200 \div 25 = \square$$

$$4 \xrightarrow{48} 25)1200$$

$$100$$

$$200$$

$$200$$

$$200$$

$$0$$

$$1200 \div 25 = 48$$

2:25 = 48:600

Therefore 48 people out of the 600 people in the barrio had tuberculosis.

2. 200:50 = 500: ( $50 \times 500$ )  $\div 200 =$   $25000 \div 200 =$ 

$$\begin{array}{r} 125\\200) \hline 25000\\ \hline 2\\5\\-4\\-10\\-10\\-0\\\end{array}$$

$$25,000 \div 200 = 125\\200:50 = 500: 125\\\end{array}$$

Therefore, 500 sheets of bond paper cost P125.

C. 1. false

Check: 15: 7 = 5: 2a : b = c : d30 ≠ 35

2. true

Check:

7	:	9	=	14	:	18
a	:	b	=	c	:	d
a	×		=		×	с
7	$\times$	18	<u>?</u>	9	$\times$	14
	5 18 <u>7</u> 126	-				

126 = 126

3. true

Check:

Check:	3	1
4: 3 = 24: 18	18	24
a: $b = c : d$	$\frac{\times 14}{72}$	$\frac{\times 3}{72}$
$a \times d = b \times c$	12	12
$4 \times 18 \stackrel{?}{=} 3 \times 24$	72 = 72	

## B. Lesson 1

Let's Try This (page 7)

A. 2. 
$$\frac{6}{6}$$
 6:6  
3.  $\frac{6}{2}$  6:2  
4.  $\frac{5}{10}$  5:10  
5.  $\frac{8}{24}$  8:24  
B. 2. 15:14  
3 5:10  
4. 15:24  
5. 2,300:5,000

Let's Try This (pages 9–10)

2. 
$$15: 10 \rightarrow 15: 10 \div 5 = 3:2$$
  
3.  $18: 21 \rightarrow \frac{18:21}{3} = 6:7$   
4. a.  $300: 150 \rightarrow \frac{300:150}{150} = 2:1$   
b.  $3: 1.5 \rightarrow 3: 1.5 \div 1.5 = 2:1$ 

Let's Try This (page 13)

Any two of the possible answers are your correct answers.

A. 1. 
$$1:5 \rightarrow a. 2 \times 1:5 = \underline{2:10}$$
  
 $\rightarrow b. 3 \times 1:5 = \underline{3:15}$   
 $\rightarrow c. 4 \times 1:5 = \underline{4:20}$   
 $\rightarrow d. 5 \times 1:5 = \underline{5:25}$   
 $\rightarrow e. 6 \times 1:5 = \underline{6:30}$ 

2. 
$$10:20 \rightarrow a.$$
  $\frac{10:20}{10} = \underline{1:2}$   
b.  $\frac{10:20}{5} = \underline{3:15}$   
c.  $\frac{10:20}{2} = \underline{5:10}$   
d.  $2 \times 10:20 = \underline{20:40}$   
e.  $3 \times 10:20 = \underline{30:60}$   
3.  $2:3 \rightarrow 2 \times 2:3 = 4:6$   
 $3 \times 2:3 = 6:9$   
 $4 \times 2:3 = 8:12$   
 $5 \times 2:3 = 10:15$   
 $6 \times 2:3 = 12:18$   
4.  $8:4 \rightarrow \frac{8:4}{4} = 2:1$   
 $\frac{8:4}{2} = 4:2$   
 $2 \times 8:4 = 16:8$   
 $3 \times 8:4 = 24:12$   
5.  $4:16 \rightarrow \frac{4:16}{4} = 1:4$   
 $\frac{4:16}{2} = 2:8$   
 $2 \times 4:16 = 8:32$   
B.  $1. 4:20$   
 $2:3 = 8:12$   
 $1:5$   
 $3:15$   
 $2. 5:10$   
 $3:15$   
 $1:2)$   
 $3. (2:3) = 8:12$   
 $10:15$   
 $4. 24:12$   
 $2:8 = 12$ 

Let's Try This (pages 15–16)

1.

Rate = 
$$\frac{\text{Cost}}{\text{Quantity}}$$
  
 $\frac{900}{50} \rightarrow \begin{array}{c} 5\% \\ 5\% \\ 50 \\ 400 \\ 400 \\ 0 \\ 0 \end{array}$ 

Therefore, the cost of the rice is P18/kilo.

2. Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$
  
 $\frac{100}{20} \rightarrow \frac{2\emptyset}{10\emptyset} \frac{5}{10\emptyset}}{\frac{10}{0}}$ 

Therefore, Bong can run 5 meters/second.

3. Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$
  
 $\frac{119}{7} \rightarrow 7)\frac{17}{119}$   
 $\frac{7}{49}$   
 $\frac{49}{0}$ 

Therefore, gasoline costs P17/liter.

4. Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$
  
 $\frac{325}{5} \rightarrow 5)\overline{325}$   
 $\frac{30}{25}$   
 $25$   
 $0$ 

Therefore, Joy can type 65 words/minute.

5. Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$
  
 $\frac{161}{3.5} \rightarrow 35.) \xrightarrow{46}_{140}$   
 $\frac{140}{210}$   
 $210$   
 $0$ 

Therefore, the jeep travels 46 km/hr.

Let's See What You Have Learned (pages 16–17)

- 1. a. 12:20
  - b. 20:12

Any two of the following possible answers are your correct answers (c and d).

c: 
$$12:20 \rightarrow \frac{12:20}{4} = \underbrace{3:5}_{2}$$
  
$$\frac{12:20}{2} = \underline{6:10}$$
$$2 \times 12:20 = 24:40$$

d. 
$$12:8 \rightarrow = 3:2$$

$$\frac{12:8}{2} = \underline{6:4}$$

$$2 \times 12 : 8 = 24 : 16$$

2. a. Rate = 
$$\frac{\text{Cost}}{\text{Quantity}}$$

$$\frac{200}{5} \longrightarrow 5)200$$

$$\frac{20}{20}$$

$$\frac{20}{0}$$

Therefore, the cost of the milk is  $\mathbb{P}40$ /liter.

b. Rate = 
$$\frac{\text{Quantity}}{\text{Price}}$$

$$\frac{52}{13} \rightarrow 13\overline{)52}$$

$$\underline{52}$$

$$0$$

Therefore, Lolit can weave 4 baskets/day.

c. Rate = 
$$\frac{\text{Cost}}{\text{Quantity}}$$

$$\frac{690}{6} \rightarrow 6)\overline{690}$$

$$\frac{6}{9}$$

$$\frac{6}{30}$$

$$\frac{30}{0}$$

Therefore, beef costs ₱115/kilo.

d. Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$
  
 $\frac{640}{8} \rightarrow \frac{8)640}{64}$   
 $\frac{64}{0}$ 

Therefore, the bus traveled 80 km/hr.

## C. Lesson 2

Let's Try This (pages 23)

- 8 2. 4:5=12:20 $4 \times 20 \stackrel{?}{=} 5 \times 12$  ( $\stackrel{?}{=}$  is read as "should be equal to")  $80 \neq 60$  ( $\neq$  is read as "is not equal to")
- 8 3. 3: 12 = 7: 20 $3 \times 20 \stackrel{?}{=} 12 \times 17$  $60 \neq 84$
- 4 4. 5:2=15:6 $5 \times 6 \stackrel{?}{=} 2 \times 15$ 30 = 30

Let's Try This (pages 26-27)

Therefore, Alex will earn P3,500 in 5 weeks.

 $(900 \times 27) \div 50 =$ 

27 × 900	$\frac{486}{5\emptyset)2430\emptyset}$
$\frac{24300}{24300}$	20
2.000	43
	40
	30
	30
	0

 $24300 \div 50 = 486$ 

Check: 50:900 = 27:486  $486 \times 50 \stackrel{?}{=} 900 \times 27$   $\frac{486}{\times 50} = \frac{27}{\times 900}$  $\frac{24300}{\times 24300} = \frac{27}{\times 900}$ 

Therefore, 27 kilos of rice will  $cost \neq 24,300$ .

Let's See What You Have Learned (page 28)

C. 1. **STEP 1** Identify the ratio in numbers.

Numbers of cookies to the grams of flour

**STEP 2** Write down the proportion in numbers.

24:200 = 36: \_\_\_\_\_ a:b = c:d

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$$

**STEP 4** Substitute the values in the formula to solve for the missing number.

Therefore, 300 grams of flour is needed to make 36 cookies. The proportion is 24 : 200 = 36 : 300.

**STEP 5** Check your answer by using the general formula:

 $a \times d = b \times c$ 

24:200 = 36:300 $24 \times 300 \stackrel{?}{=} 200 \times 36$ 

2. **STEP 1** Identify the ratio in words.

a scale of cm to km

**STEP 2** Write down the proportion in numbers.

$$2:3=5:$$
  
 $a:b=c:d$ 

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$$

**STEP 4** Substitute the values in the formula to solve for the missing number.

Therefore, the distance between the two towns is 7.5 km. The proportion is: 2:3=5:7:5

**STEP 5** Check your answer by using the general formula:

$$2: 3 = 5: 7.5$$
  

$$2 \times 7.5 = 3 \times 5$$
  

$$\frac{7.5}{15.0} = \frac{\times 5}{15}$$

3. **STEP 1** Identify the ratio in words.

The number of people who preferred Brand A to the number of people who preferred Brand B.

## **STEP 2** Write the proportion in numbers.

```
3:5=27:
a:b=c:d
```

**STEP 3** Determine the formula to be used in finding the missing number.

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$$

**STEP 4** Substitute the values in the formula to solve for the missing number.

$$(5 \times 27) \div 3 =$$

 $135 \div 3 = 45$ 

Therefore, if 27 people preferred brand A, 45 people preferred brand B. The proportion is: 3:5 = 27:45

**STEP 5** Check your answer by using the general formula:

 $a \times d = b \times c$ 

3:5=27:45 $3 \times 45 \stackrel{?}{=} 5 \times 27$ 

45		27	
<u>× 3</u>		$\times$	5
135	=	1	35

## D. What Have You Learned? (pages 32–34)

1. a. 36:64 or  $^{36}/_{64}$ 

Simplifying the ratio we have:

$$\frac{36:64}{4} = 9:16$$

The ratio of the number of satisfied respondents to the number of dissatisfied respondents is *nine is to sixteen*.

b. 64:36 or  $\frac{64}{36}$ 

Simplifying the ratio we have:

$$\frac{64:36}{4}$$
=16:9

The ratio of the number of dissatisfied respondents to the number of satisfied respondents is *sixteen is to nine*.

c. 64:100 or  $^{64}/_{100}$ 

Simplifying the ratio we have:

$$\frac{64:100}{4} = 16:25$$

The ratio of the number of dissatisfied respondents to the total number of respondents is *sixteen is to twenty five*.

d. 36:100 or  $^{36}/_{100}$ 

Simplifying the ratio we have:

$$\frac{36:100}{4} = 9:25$$

The ratio of the number of dissatisfied respondents to the total number of respondents is *nine is to twenty five*.

2. Any two of the possible answers are correct answers.

a. 
$$\frac{6:15}{3} = \underline{3:5}$$

$$2 \times (6:15) = \underline{12:30}$$

$$3 \times (6:15) = \underline{18:45}$$
b. 
$$\frac{20:25}{5} = \underline{4:5}$$

$$2 \times (20:25) = \underline{40:50}$$

$$3 \times 20:25 = \underline{60:75}$$
c. 
$$2 \times (8:1) = \underline{16:2}$$

$$3 \times (8:1) = \underline{24:3}$$

$$4 \times (8:1) = \underline{32:4}$$

3. a. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

72:3 or 
$$\frac{72}{3}$$

**STEP 2** Solve for the rate:

Rate = 
$$\frac{Cost}{Quantity}$$
  
 $3\overline{)72}$   
 $\frac{6}{12}$   
 $12$   
 $0$ 

Therefore, the rate is ₱24/kilo of mangoes.

b. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

$$390:6 \text{ or } \frac{390}{6}$$

**STEP 2** Solve for the rate.

Rate = 
$$\frac{\text{Quantity}}{\text{Time}}$$

Therefore, the secretary can type 65 words per minute.

c. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

66.50:7 or 
$$\frac{66.50}{7}$$

**STEP 2** Solve for the rate.

$$Rate = \frac{Cost}{Quantity}$$

$$7 \overline{\smash{\big)}66.50}$$

$$63$$

$$35$$

$$35$$

$$0$$

Therefore, the long distance call costs \$9.50/ minute.

d. **STEP 1** Express the two quantities in ratio form.

Expressing this in ratio form we have:

$$117:9 \text{ or } \frac{117}{9}$$

**STEP 2** Solve for the rate.

$$Rate = \frac{Quantity}{Time}$$

$$9)117$$

$$9)117$$

$$9$$

$$27$$

$$27$$

$$0$$

Therefore, the motorcycle traveled 13 meters per second.

B. 1. **STEP 1** Identify the ratio in words.

Buckets of cement to the buckets of sand

**STEP 2** Write down the proportion in numbers.

$$1:3 = 4.5:$$
 a : b = c : d

**STEP 3** Determine the formula to be used to find the missing number.

$$(\mathbf{b} \times \mathbf{c}) \div \mathbf{a} = \mathbf{d}$$

**STEP 4** Substitute the values in the formula and solve for the missing number.

$$(3 \times 4.5) \div 1 = \square$$
4.5
$$3 \times 4.5 \times 1 = 13.5$$

$$\times 3$$
13.5

Therefore, Mang Pedro needs 13.5 buckets of sand. The proportion is: 1:3 = 4.5:13.5

**STEP 5** Check your answer by using the general equation:

 $a \times d = b \times c$ 

1.3 = 4.5 : 13.5 $1 \times 13.5 = 3 \times 4.5$ 13.5 = 13.5

2. **STEP 1** Identify the ratio in words.

Number of students who passed the Math exam to the total number of students

**STEP 2** Write down the proportion in numbers.

$$4:5 = \square:40$$
$$a:b = c:d$$

$$(a \times d) \div b = c$$

**STEP 4** Substitute the values in the formula to solve for the missing number.

$$(4 \times 40) \div 5 = \boxed{}$$

$$40$$

$$\times 4$$

$$160 \quad 160 \div 5 = \boxed{}$$

$$5)160$$

$$15$$

$$10$$
The proportion is:  $4:5 = 32:40$ 

**STEP 5** Check your answer by using the general equation:

$$a \times d = b \times c$$

$$4:5=32:40$$

$$4 \times 40 = 5 \times 32$$

$$32 \qquad 40$$

$$\times \frac{5}{160} \qquad \frac{\times 4}{160}$$

$$160 = 160$$

Therefore, 32 students out of 40 passed the math exam.

3. **STEP 1** Identify the ratio in words.

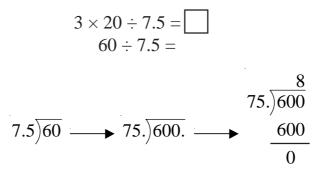
Number of dresses to the number of meters of clothing material

**STEP 2** Write down the proportion in numbers.

$$3:7.5 = : 20$$
  
 $a:b=c:d$ 

$$(a \times d) \div b = c$$

**STEP 4** Substitute the values in the formula and solve for the missing number.



The proportion is: 3:7.5 = 8:20

Therefore, 20 meters of cloth makes 8 dresses.

**STEP 5** Check your answer by using the general equation.

 $a \times d = b \times c$ 

3:7.5 = 8:20 $3 \times 20 \stackrel{?}{=} 7.5 \times 8$  $\frac{7.5}{\times 8} = \frac{20}{\times 3} \\ \frac{8}{60.0} = \frac{20}{60}$ 



- Brownstein, S. C. *Barron's How to Prepare for the Scholastic Aptitude* <u>Test</u>. Manila: Global Publishing. 1991.
- 1991.Cariño, I. D. *General Mathematics for High School 1*. Pasig City: Anvil Publishing, Inc. 1999.